Një kod i vogël MATLAB për mësimdhënien e programimit linear

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Përmbajta

1. Synime
2. Kodi MATLAB
3. Shembuj problemesh të zgjidhura
4. Burime biblografike

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Synime

1. Shmangje e ngarkesës komputacionale, përqëndrim tek idetë.
2. Analizë kritike e algoritmit simpleks, eksperimentim me probleme jotriviale.
3. Modifikime, zgjerime, alternativa të procedurës dhe/ose simpleksit

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Kodi MATLAB

```
function   A = celes(A,r,s)


[m,n]=size(A);
for  i=1:m
     if  i==r
          A(r,:) =  A(r,:)/A(r,s);
     else
          A(i,:)= A(i,:) - (A(i,s)/A(r,s))*A(r,:);
     end
end
end
```

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Shembull

Të zgjidhet problemi i programimit linear

$$\min \quad 6x_1 + 3x_2 + 4x_3$$

$$\begin{cases} x_1 & & & & \geq & 30 \\ & x_2 & & & \leq & 50 \\ & & & x_3 & \geq & 20 \\ x_1 & + & x_2 & + & x_3 = & 120 \end{cases}$$

$$x_1, x_2, x_3 \geq 0.$$

Me metodën e dy fazave. Tre ndryshore shtesë/tepricë dhe tre artificiale. Futet tabela e problemit ndihmës në MATLAB:

```
B=[1 0 0 -1 0 0 1 0 0 30; 0 2 0 0 1 0 0 0 0 50;
0 0 1 0 0 -1 0 1 0 20; 1 1 1 0 0 0 0 0 1 120;
0 0 0 0 0 0 1 1 1 0]
B=sym(B); latex(B)
```

Reduktimi në trajtë kanonike

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 120 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Veprime me rreshtat: V $\mapsto$ V - (I + III +IV)

```
B=celes(B,1,7);
B=celes(B,3,8);
B=celes(B,4,9) latex(B)
```

Përftohet tabela

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 120 \\ -2 & -1 & -2 & 1 & 0 & 1 & 0 & 0 & 0 & -170 \end{pmatrix}.$$

Simpleksi me dy faza: faza I

Problemi në trajtë kanonike, fillojmë me simpleksin.

B=celes(B,3,3)
latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 100 \\ -2 & -1 & 0 & 1 & 0 & -1 & 0 & 2 & 0 & -130 \end{pmatrix}.$$

B=celes(B,1,1)
latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 0 & 1 & 0 & 1 & 0 & 1 & -1 & -1 & 1 & 70 \\ 0 & -1 & 0 & -1 & 0 & -1 & 2 & 2 & 0 & -70 \end{pmatrix}.$$

B=celes(B,4,2)
latex(B)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 100 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 0 & 1 & 0 & 1 & 0 & 1 & -1 & -1 & 1 & 70 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Simpleksi me dy faza: faza II

A=B; A(:,[7,8,9])=[];
A(5,:)=[6 3 4 0 0 0 0]
latex(A)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 100 \\ 0 & 2 & 0 & 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 1 & 0 & 1 & 0 & 1 & 70 \\ 6 & 3 & 4 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

A=celes(A,1,1);
A=celes(A,3,3)
latex(A)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 100 \\ 0 & 2 & 0 & 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 1 & 0 & 1 & 0 & 1 & 70 \\ 0 & -3 & 0 & 0 & 0 & -2 & -680 \end{pmatrix}.$$

Tani fillojmë me zbatimin e simpleksit:

A=celes(A,2,2)
latex(A)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -\frac{1}{2} & 1 & 75 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 25 \\ 0 & 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & 45 \\ 0 & 0 & 0 & 0 & \frac{3}{2} & -2 & -605 \end{pmatrix}.$$

Simpleksi me dy faza: faza II (vazhdim)

A=celes(A,2,2)
latex(A)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -\frac{1}{2} & 1 & 75 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 25 \\ 0 & 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & 45 \\ 0 & 0 & 0 & 0 & \frac{3}{2} & -2 & -605 \end{pmatrix}.$$

A=celes(A,4,6)
latex(A)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 25 \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & 0 & 65 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & 45 \\ 0 & 0 & 0 & 2 & \frac{1}{2} & 0 & -515 \end{pmatrix}.$$

Tabelë “optimale” (koef. e reduktuar të f.q. > 0);
lexojmë të vetmen zgjidhje optimale të problemit:
$x_1 = 30$, $x_2 = 25$, $x_3 = 65$. Vlera optimale (minimale) e f.q. $= 515$.

Me metodën e numrit M 1/3

Me komandat syms M

B=[1 0 0 -1 0 0 1 0 0 30; 0 2 0 0 1 0 0 0 0 50; 0 0 1 0 0 -1 0 1 0 20; 1 1 1 0 0 0 0 0 1 120; 6 3 4 0 0 0 M M M 0]

latex(B) Tabela e parë

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 120 \\ 6 & 3 & 4 & 0 & 0 & 0 & M & M & M & 0 \end{pmatrix}$$

E sjellim f.q. në trajtë kanonike:

B=celes(B,1,7);
B=celes(B,3,8);
B=celes(B,4,9)
latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 120 \\ 6-2M & 3-M & 4-2M & M & 0 & M & 0 & 0 & 0 & -170M \end{pmatrix}$$

Me metodën e numrit M 2/3

B=celes(B,3,3) latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 100 \\ 6-2M & 3-M & 0 & M & 0 & 4-M & 0 & 2M-4 & 0 & -130M-80 \end{pmatrix}$$

B=celes(B,1,1) latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 0 & 1 & 0 & 1 & 0 & 1 & -1 & -1 & 1 & 70 \\ 0 & 3-M & 0 & 6-M & 0 & 4-M & 2M-6 & 2M-4 & 0 & -70M-260 \end{pmatrix}$$

B=celes(B,2,2) latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 25 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & -1 & -1 & 1 & 45 \\ 0 & 0 & 0 & 6-M & \frac{M}{2}-\frac{3}{2} & 4-M & 2M-6 & 2M-4 & 0 & -45M-335 \end{pmatrix}$$

Me metodën e numrit M 3/3

B=celes(B,4,6)
latex(B)

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 25 \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & 0 & -1 & 0 & 1 & 65 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & -1 & -1 & 1 & 45 \\ 0 & 0 & 0 & 2 & \frac{1}{2} & 0 & M-2 & M & M-4 & -515 \end{pmatrix}$$

Tabela optimale.
Lexojmë të njëjtën zgjidhje optimale si më parë.

Me metodën e simpleksit dual

Kod Matlab

```
A=[1 0 0 -1 0 0 30; 0 2
0 0 1 0 50; 0 0 1 0 0
-1 20; 1 1 1 0 0 0 120;
6 3 4 0 0 0 0]
A=sym(A)
latex(A)
```

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 30 \\ 0 & 2 & 0 & 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 0 & 0 & -1 & 20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 120 \\ 6 & 3 & 4 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
A([1,3],:)=-A([1,3],:)
A=celes(A,4,2)
latex(A)
```

$$\begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & -30 \\ -2 & 0 & -2 & 0 & 1 & 0 & -190 \\ 0 & 0 & -1 & 0 & 0 & 1 & -20 \\ 1 & 1 & 1 & 0 & 0 & 0 & 120 \\ 3 & 0 & 1 & 0 & 0 & 0 & -360 \end{pmatrix}$$

Me metodën e simpleksit dual (vazhdim)

Fillojmë me zbatimin e algoritmit simpleks:

```
A=celes(A,2,3)
latex(A)
```

$$\begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 & -30 \\ 1 & 0 & 1 & 0 & -1/2 & 0 & 95 \\ 1 & 0 & 0 & 0 & -1/2 & 1 & 75 \\ 0 & 1 & 0 & 0 & 1/2 & 0 & 25 \\ 2 & 0 & 0 & 0 & 1/2 & 0 & -455 \end{pmatrix}$$

```
A=celes(A,1,1)
latex(A)
```

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 30 \\ 0 & 0 & 1 & 1 & -\frac{1}{2} & 0 & 65 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & 45 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & 25 \\ 0 & 0 & 0 & 2 & \frac{1}{2} & 0 & -515 \end{pmatrix}$$

Tabelë përfundimtare.

Gjejmë sërisht zgjidhjen optimale.

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Burime bibliografike

[1] D. Hill, A Tool for Teaching Linear Programming within MATLAB, The College Mathematics Journal, Vol. 21, No. 1 (Jan., 1990), pp. 55-56.

[2] D. Hill, Experiments in Computational Matrix Algebra, Random House, New York, 1988.

[3] B. Kolman, Introductory Linear Algebra with Applications, Fourth Edition, Macmillan, New York, 1988.

[4] C. Moler, J. Little, S. Bangert, S. Kleiman, MATLAB User's Guide, The MathWorks, Inc., Suite 250, 20 North Main St., Sherborn, MA 01770, 1987.

Faleminderit për vëmendjen!