Convexity along vector fields and extremal points

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The notion of convexity along vector fields was introduced recently by M. Bardi and F. Dragoni [1], [2] and it widely generalises various notions of convexity given by several authors in various contexts of homogeneous Lie groups or Carnot-Carathódory groups. In line with the fact that usual convexity is a "one-dimensional" property, it amounts to the simple requirement that a function should be convex along the trajectories of the linear combinations of a given system of vector fields and requires no algebraic structure on the domain of definition of the function. Clearly, usual convexity in euclidean spaces corresponds in this setting to convexity along the canonical directions of the space. I will speak about a result that–in line with its classical analog–states that first-order necessary conditions for optimality are sufficient as well: more precisely, if (i) a function is convex along a given finite system of vector fields, (ii) its horizontal gradient with respect to these fields vanishes at some given point, and (iii) the underlying domain of definition of the function is connected by trajectories of these same vector fields, then the said point is a global minimum of the function.

References

[1] M. Bardi and F. Dragoni, *Convexity and semiconvexity along vector fields*, Calculus of Variations and Partial Differential Equations, **42**, 3-4 (2011) 405-427.

[2] M. Bardi and F. Dragoni, *Subdifferential and properties of convex functions with respect to vector fields*, Journal of Convex Analysis, **21** 3 (2014) 785-810.